

Constraining phases of quark matter with studies of r -mode damping in neutron stars

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Abstract

The r -mode instability in rotating compact stars is used to constrain the phase of matter at high density. The color-flavor-locked phase with kaon condensation (CFL-K0) and without (CFL) is considered in the temperature range $10^8\text{K} \lesssim T \lesssim 10^{11}\text{K}$. While the bulk viscosity in either phase is only effective at damping the r -mode at temperatures $T \gtrsim 10^{11}\text{K}$, the shear viscosity in the CFL-K0 phase is the only effective damping agent all the way down to temperatures $T \gtrsim 10^8\text{K}$ characteristic of cooling neutron stars. However, it cannot keep the star from becoming unstable to gravitational wave emission for rotation frequencies $\nu \approx 56 - 11\text{Hz}$ at $T \approx 10^8 - 10^9\text{K}$. Stars composed almost entirely of CFL or CFL-K0 matter are ruled out by observation of rapidly rotating neutron stars, indicating that dissipation at the quark-hadron interface or nuclear crust interface must play a key role in damping the instability.

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I. INTRODUCTION

The r -mode instability of rotating compact stars is of wide astrophysical interest for several reasons [1]. The majority of neutron stars have rotation period $P \sim 0.5$ seconds, much slower than theoretical limits: $P \sim 0.001$ s ($\nu = 1/P \sim 1$ kHz) [2]. Even the fastest spinning neutron star PSR J1748-2446ad with $\nu = 716$ Hz [3] is not near the theoretical stability limit, with the majority of “spun-up” neutron stars in low mass X-ray binaries (LMXBs) catalogued between 300-640 Hz. The r -mode provides a mechanism for spinning down young neutron stars [4] and limiting spin-up frequencies in millisecond pulsars (see eg. [5] for a review) through angular momentum loss in gravitational wave emissions. These gravitational waves may be detected given the expected improved sensitivities of ground-based interferometers eg. VIRGO, advanced LIGO [6]. In addition, r -modes are probes of the equation of state of cold and dense matter through viscous damping effects and advance our understanding of nuclear and quark interactions by providing an astrophysical and phenomenological link to Quantum Chromodynamics (QCD) over a certain range of temperatures ($10^7 \text{ K} \lesssim T \lesssim 10^{11} \text{ K}$) and at super-saturation densities [$n_B \sim (2-5)n_{\text{sat}}$]. Only a few studies of r -modes linking them to phenomenology of quark matter in neutron stars have been carried out thus far [7–11]. In this work we use r -mode damping to constrain the presence of a realistic phase of superconducting quark matter [12, 13] in the core of neutron stars: viz., one that includes the effects of a sizable strange quark mass m_s at intermediate baryon density.

A key parameter in understanding the r -mode instability in rotating compact stars is the critical frequency Ω_c for the onset of the instability ($\Omega = 2\pi/P$ rad/s). Gravitational radiation tends to make the r -mode grow on a characteristic time scale τ_{GR} . Internal friction such as viscosity or mutual friction tends to damp r -mode growth on a characteristic time scale τ_F . The competition between the two determines Ω_c at any temperature. At $\Omega \geq \Omega_c$, the r -mode develops an instability ($1/\tau_{\text{total}} = 1/\tau_{GR} + 1/\tau_F < 0$). Previous works [9–11] have studied the r -mode damping in normal (ungapped) quark matter as well as some superconducting (gapped) phases such as the color-flavor-locked phase (CFL), and found that bulk and shear viscosity in the fully gapped phase is large enough to render rapidly spinning quark stars ($\Omega \geq 0.1\Omega_{\text{Kepler}}$) stable. However, this conclusion applies only in a restricted temperature regime $T \gtrsim 10^{10} \text{ K}$, since the shearing mean free path due to phonons otherwise becomes larger than the star’s radius ($\sim 10 \text{ km}$). Furthermore, for temperatures $T \lesssim 10^8 \text{ K}$, mutual friction in the CFL phase has been shown to be too weak to damp r -modes [11], arguing for rapid spin-down of CFL stars to less than a few Hz, effectively ruling out cold CFL matter in any rapidly rotating neutron stars. The r -mode damping in the temperature regime $10^8 \text{ K} \lesssim T \lesssim 10^{10} \text{ K}$ for color-superconducting phases of quark matter is therefore an open question, and is the focus of this work. Here we study r -mode damping and the critical frequency curve for a neutron star made up mostly or entirely of the kaon condensed CFL (CFL-K0) phase.

The CFL phase with symmetric pairing of up-, down- and strange-quarks is severely stressed at realistic chemical potential $\mu_q \sim 300 \text{ MeV}$ because the strange quark mass $m_s \sim 100 \text{ MeV}$ is non-negligible and costs extra energy $m_s^2/(2\mu_q)$ compared to the much lighter up- and down-quarks. In Ref. [14], it was shown that it is energetically favorable for the CFL vacuum to be in a rotated chiral state with a non-zero kaon condensate when the cost $m_s^2/(2\mu_q)$ exceeds the kaon mass m_K , the lightest meson in high-density QCD [15]. Realistic neutron star densities could support the CFL-K0 phase. We find that in the CFL-K0 phase, while the bulk viscosity has little role to play in the temperature region relevant to r -mode damping in cooling neutron stars, the shear viscosity from kaons in the CFL-K0 phase is important even below $T \lesssim 10^9 \text{ K}$, unlike the CFL phase. In essence, this fact coupled with the large mean free path of the phonons controls the main features of the critical frequency curve (Fig. 2).

II. THE r -MODE

r -mode is a pulsation mode intimately linked to the rotational properties of the star, and the restoring force here is the Coriolis force. The frequency of r -mode oscillations depends on the star's rotation frequency as well as the average density of the star. Including second-order rotational effects, the mode frequency ω_r in the inertial frame is [1, 16]

$$\omega_r = \omega_{rot} - m\Omega \approx \left[\frac{2}{m+1} - m \right] \Omega + \kappa_2 \frac{\Omega^2}{\pi G \bar{\rho}_0} + O(\Omega^4), \quad (1)$$

where ω_{rot} is the mode frequency in the co-rotating frame and m is the azimuthal quantum number of the mode, taken to be 2 for the first r -mode to become unstable (higher m modes become unstable at higher frequency). The number κ_2 includes the sensitivity to the density profile (see Refs. [9, 17]), $\bar{\rho}_0$ is the average density of the unperturbed star and G is Newton's constant.

The r -mode couples to gravitational waves through the current multipole of the perturbation [1]. All modes with azimuthal quantum number $m \geq 2$ suffer the so-called CFS instability [18, 19] when $\Omega > \omega_{rot}/m$ so that the r -mode energy grows at the expense of the star's rotational kinetic energy as angular momentum is lost to gravitational wave emission. In general, viscous forces counter this energy growth. The timescale τ associated with growth or dissipation is given by $1/\tau_i = -\frac{1}{2E} \left(\frac{dE}{dt} \right)_i$, where the subscript i corresponds to various dissipative forces. Details about computing E and $\left(\frac{dE}{dt} \right)_i$ are given in Refs. [9, 17]. Explicitly, the gravitational radiation timescale is [1]

$$\frac{1}{\tau_{GW}} = -\frac{32\pi G}{c} \frac{(m-1)^{2m}}{[(2m+1)!!]^2} \int_0^R dr \rho \left[r \frac{\Omega}{c} \frac{m+2}{m+1} \right]^{2m+2}. \quad (2)$$

This timescale is negative, indicating exponential mode growth (the instability).

Viscosity (bulk, shear and otherwise) is a source of damping for the r -mode. For a given phase with shear viscosity η , the shear viscosity timescale is [1]

$$\frac{1}{\tau_\eta} = \frac{(m-1)(2m+1)}{\int_0^R dr \rho r^{2m+2}} \int_0^R dr \eta r^{2m}. \quad (3)$$

We find that the bulk viscosity damping timescale plays a minor role in our calculation, see Table I and Figs. 1, 2.

Mutual friction is another source of dissipation for the r -mode energy. It occurs due to the scattering of phonons from quantized vortices in the superfluid component. For the CFL phase, this has been calculated to be [11]: $\frac{1}{\tau_{MF}} \simeq 36.2 \left(\frac{T}{\mu_q} \right)^5 \Omega$ in the limit of negligible vortex mass. The critical rotation frequency Ω_c of the star can be determined by the criterion that at this frequency, the fraction of energy dissipated per unit time exactly cancels the fraction of energy fed into the r -mode by gravitational wave emission:

$$\frac{1}{\tau_{total}} \Big|_{\Omega_c} \equiv \left[\frac{1}{\tau_{GW}} + \frac{1}{\tau_\eta} + \frac{1}{\tau_\zeta} + \frac{1}{\tau_{MF}} \right] \Big|_{\Omega_c} = 0. \quad (4)$$

Stable rotation frequencies at any temperature are limited by the smaller of the critical frequency as computed from Eq. (4) or the Kepler limit $\Omega/\Omega_K=1$. As depicted in Fig. 2, the region above the temperature-dependent Ω_c curve is unstable to r -mode oscillations and the star upon entering this region will be spun down rapidly to $\Omega < \Omega_c$ by emitting gravitational waves and losing its angular momentum in the process.

III. CFL-K0: EOS AND VISCOSITY

For simplicity, we adopt a homogeneous composition assuming that the entire star is made of CFL-K0 matter. This allows a direct comparison with r -mode calculations in hadronic matter with and without a crust [20]. We will see later that a crust is required for agreement with observations, but the more elementary case of a pure CFL-K0 star should be tested first. Just as for the CFL phase, the neutral kaons are expected to dominate over the phonons in determining the bulk viscosity. However, the condensation changes the temperature dependence of the viscosity, which is the most important outcome for the r -mode damping. The pressure of the CFL-K0 phase, including terms up to $O(m_s^4)$ is given by [14, 21]

$$P_{\text{CFL-K0}} = P_{\text{CFL}} + \frac{1}{2} f_\pi^2 \left(\frac{m_s^2}{2k_f} \right)^2, \quad (5)$$

The pressure from the Nambu-Goldstone bosons introduces a negligible temperature dependence. We use the perturbative expression for f_π [15]. This EoS differs from the CFL EoS only at $O(m_s^4)$. In the above expression, the common Fermi momenta k_F also determines the CFL pressure [22]

$$P_{\text{CFL}} = -\frac{3}{\pi^2} \sum_{i=u,d,s} \int_0^{k_F} dk k^2 \left[\sqrt{k^2 + m_i^2} - \mu \right] + \frac{3\Delta^2 \mu^2}{\pi^2} - B, \quad (6)$$

where Δ is the superconducting quark gap and B is the MIT Bag constant. We choose $\Delta = 100$ MeV and $B = 80$ MeV/fm³ such that the energy per baryon $E/A < 930$ MeV at zero pressure. In the limit of vanishing up and down quark mass, we have up to $O(m_s^4)$: $k_F = \frac{1}{3} \left(6\mu - \sqrt{9\mu^2 + 3m_s^2} \right) \approx \mu - m_s^2/(6\mu) + m_s^4/(72\mu^3)$.

Evaluating the expression for $P_{\text{CFL-K0}}$ pressure using Eq.(6), we obtain

$$P_{\text{CFL-K0}} \approx \frac{3\mu^4}{4\pi^2} - B + \frac{3\mu^2}{4\pi^2} (4\Delta^2 - m_s^2) + \frac{m_s^4}{32\pi^2} \left(1 + 6 \ln \frac{4\mu^2}{m_s^2} + 4\pi^2 \bar{f}_\pi^2 \right), \quad (7)$$

where $\bar{f}_\pi^2 \equiv f_\pi^2/k_F^2 \approx (21 - 8 \ln 2)/(36\pi^2)$. The energy density then follows from standard thermodynamic relation $\epsilon = -P_{\text{CFL-K0}} + n\mu$ with number density $n = \partial_\mu P_{\text{CFL-K0}}$. Eliminating μ between pressure and energy density, we get the EoS in the useful form

$$P_{\text{CFL-K0}} \approx \frac{1}{3} (\epsilon - 4B) + \frac{4\Delta^2 - m_s^2}{3\pi} \sqrt{\epsilon - B} - \frac{m_s^4}{12\pi^2} \left[1 + \left(1 - \frac{4\Delta^2}{m_s^2} \right)^2 - 2\pi^2 \bar{f}_\pi^2 - 3 \ln \frac{8\pi \sqrt{\epsilon - B}}{3m_s^2} \right]. \quad (8)$$

The dominant contribution to the bulk viscosity in the fully gapped CFL or CFL-K0 phase is determined by weak reactions involving the lightest modes [23, 24]. In the CFL-K0 phase, this is the neutral kaons, a fraction of which are in the condensate. From Alford et. al [24], bulk viscosity is

$$\zeta = \frac{n^2}{\chi^2} \frac{\lambda}{\omega^2 + \lambda^2/\chi^2}, \quad (9)$$

where the kaon number density n , susceptibility χ and weak equilibration rate for kaons in the condensate are computed in Ref. [24] using the 2PI formalism in order to assess thermal effects consistently. Note that $\delta m = m_K - \mu_K < 0$ for the condensed phase but $\delta m > 0$ for the pure CFL phase. For the typical parameter values [9, 23] used in Fig. 1, $T_c \sim 3 \times 10^{11} - 4 \times 10^{11}$ K. There is a negligible contribution to bulk viscosity due to phonon scattering above $T \gtrsim 5 \times 10^8$ K [25].

Kaon condensation breaks flavor symmetry, and the associated Nambu-Goldstone mode contributes to shear viscosity at low temperature. Although the kaonic contribution is smaller than that from the phonons, it becomes important when the phonon mean free path exceeds the stellar radius at $T \lesssim 10^{10}$ K - see Fig 1. The leading order contribution was calculated in terms of an unknown dimensionless coefficient C [26]. Analytic expressions for the shear viscosity could be derived for very small or large value of the coupling C :

$$\begin{aligned} C \ll 1: \quad \eta &\approx 3.44 \times 10^{-4} \frac{v^{11}}{C^2} \frac{f_\pi^4 \Delta^4}{T^5}, \\ C \gg 1: \quad \eta &\approx 1.7 \times 10^{-8} \frac{v^{11}}{C^4} \frac{f_\pi^4 \Delta^8}{\sin^4(\phi/f_\pi) \mu_K^4 T^5}. \end{aligned} \quad (10)$$

For natural values $C \sim 1$, shear viscosity η has to be calculated numerically which we have done and cross-checked against the results of Ref. [26]. Here also η shows a T^{-5} dependence similar to the contribution from massless phonon scattering. Phonon contribution to shear viscosity is

$$\eta_{\text{Phonon}} = 3.745 \times 10^6 \left(\frac{\mu_q}{\text{MeV}} \right)^8 T_9^{-5} \text{g}/(\text{cm s}), \quad (11)$$

where T_9 is the temperature in units of 10^9 K. The effects of mutual friction have been shown to be too small to effectively damp the r -mode instability at $T \lesssim 10^8$ K [11]. In this work, we include the mutual friction damping.

In Fig. 1 below, we show the bulk and shear viscosity from the CFL-K0 phase in comparison to the pure CFL phase.

IV. DAMPING EFFECTS

Next, we present results for the damping times from the various mechanisms discussed previously and obtain the critical frequency plot. The damping times are listed in Table I. The bulk

TABLE I. Damping times at $T = 10^9$ K for the CFL-K0 phase, the CFL phase *with* $O(m_s^4)$ corrections (labeled CFL4) and *without*. For the CFL-K0 phase, the shear damping timescale due to kaons with parameter value $C = 1$ (see text) is shown. The central energy density ρ_c (in units of nuclear saturation density $\rho_0 = 2.54 \times 10^{14} \text{g}/\text{cm}^3$) is chosen to yield a $1.4 M_\odot$ star. The stellar rotation frequency is taken to be half the Kepler frequency $\Omega_K = \frac{4}{9} \sqrt{2\pi G \bar{\rho}}$.

EoS	ρ_c/ρ_0	R (km)	Ω_K (rad/s)	τ_ζ (s)	τ_η (s)	τ_{GW} (s)
CFL-K0	2.526	10.748	6696	9.6×10^{22}	4.9×10^{13}	-7.6×10^2
CFL4	2.526	10.709	6697	1.7×10^{12}	—	-7.8×10^2
CFL	2.526	10.729	6696	1.7×10^{12}	—	-7.7×10^2

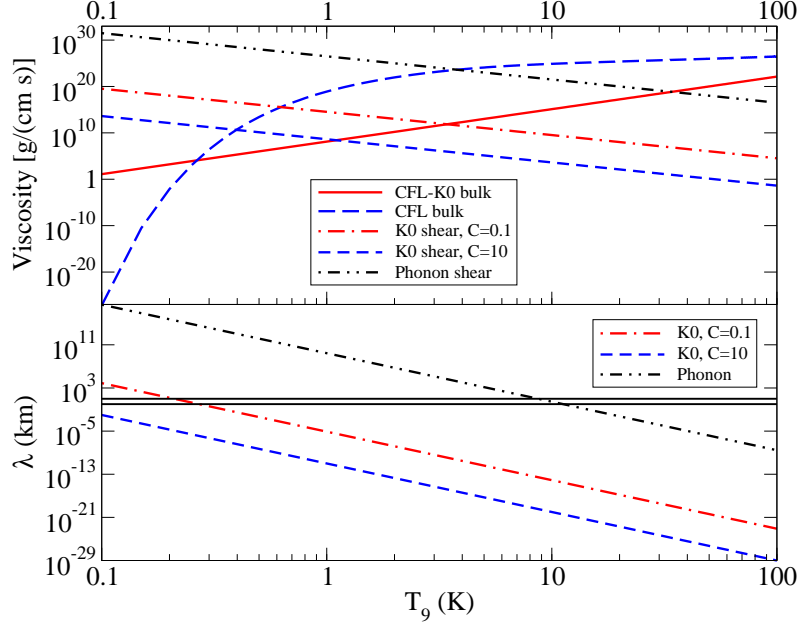


FIG. 1. Top panel: viscosities for the CFL and CFL-K0 phase. Bottom panel: phonon and kaon shear viscosity mean free path λ . The typical parameter values [9, 23] are $\mu_q = 310$ MeV, $\mu_k = 17.92$ MeV and $\omega = 2\pi \text{ ms}^{-1}$, $\delta m = 1$ MeV for CFL, $\delta m = -1$ MeV for CFL-K0. In the bottom panel, the crossings with the solid (black) horizontal lines at 1km and 10km indicate when λ is on the order of the stellar size, making the shear viscosity irrelevant.

viscosity of the CFL-K0 phase is smaller than that of the CFL or ungapped phase for temperatures $T \lesssim 10^{11}$ K. Consequently, bulk viscosity damping timescales are much longer than the typical timescale for gravitational wave growth, rendering bulk viscosity irrelevant to r -mode damping in cooling neutron stars. The damping timescale from shear viscosity in the CFL-K0 phase becomes small enough to effectively damp the r -mode as temperatures fall to $T \sim 5 \times 10^{10}$ K. However, this is still dominated by the phonons, since the kaon shear viscosity is much smaller. The phonon mean free path increases with decreasing T , therefore, phonon shearing collisions occur less often. The mean free path λ_ϕ is estimated from the kinetic theory relation $\lambda_\phi \sim \eta / (n_\phi p_\phi)$ using phonon density n_ϕ and thermal momentum p_ϕ , see Refs. [9, 26, 27]. In Fig. 1, we show the mean free path of the phonons and kaons. A larger value of the parameter C corresponds to a larger cross-section for shearing collisions and therefore yields a smaller mean free path at the same temperature. It can be seen from Fig. 1 that kaon shear viscosity is important in the temperature range $10^8 \text{ K} \lesssim T \lesssim 10^{10} \text{ K}$, where phonon shear viscosity is no longer important.

Finally, we compute the critical frequency curve for the CFL-K0 phase according to Eq.(4). We notice two “dips” in the critical frequency curve: the first one at $T \sim 8 \times 10^9$ K, when the phonon mean free path λ_ϕ becomes larger than the stellar radius, and the second at $T \sim 2.5 \times 10^8$ when the kaon mean free path becomes larger. This second dip is sensitive to the value of C , appearing earlier for smaller C . In order to obtain a smooth behaviour for the critical frequency curve, we use for convenience a sharply peaked cutoff function $f(\lambda_\phi) = [1 - \tanh(\lambda_\phi/\text{km} - 5)]/2$ centered around a 5 km stellar radial distance. The maximum stable frequency accommodated by the CFL-K0 phase falls well below the observed bound on rotation rates in LMXBs.

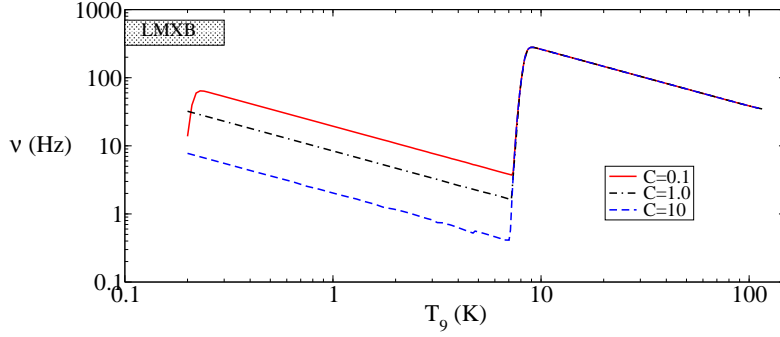


FIG. 2. The critical frequency ν as a function of temperature for CFL-K0 quark stars. The curves end abruptly when the shear mean free path exceeds the stellar radius. We use the same parameters as in Fig. 1. The box represents typical temperatures [28] (2×10^7 - 3×10^8 K) and rotation frequencies (300-700 Hz) of the majority of observed neutron stars in LMXBs.

V. CONCLUSIONS

r -mode oscillations of compact stars in the kaon-condensed CFL phase are considered. The mode frequency is almost exactly the same as that for a CFL star [9] since the equation of state for the CFL-K0 phase only differs at $O(m_S^4)$. The mode growth and viscous damping timescales in the temperature range $10^8 \text{ K} \lesssim T \lesssim 10^{11} \text{ K}$ is studied based on the dominant bulk and shear viscosity contributions. Compared to the pure CFL phase, the bulk viscosity in CFL-K0 is smaller and does not play a significant role. At temperatures $T \gtrsim 10^{10} \text{ K}$ shear viscosity associated with phonon scattering is important and determines the temperature dependence of the critical frequency curve. At lower temperatures, the phonon mean free path exceeds the star radius, so that the new mechanism for shear viscosity associated with kaon condensation [26] provides the dominant dissipation. In comparison to pure CFL stars, CFL-K0 stars are more stable against the r -mode instability at the lower end of the temperature range considered. However, given the observed LMXB spin rates, the critical frequency curve predicts that even pure CFL-K0 stars are unlikely to exist. Just as in the neutron matter case, viscous damping just beneath the core-crust interface (Ekman layer) appears to be necessary to provide a large enough stable rotation rate consistent with LMXB data [20].

The main result of the current work is that given the known dissipative mechanism in CFL or CFL-K0 phase, a pure quark star with these phases is ruled out by observed LMXB spin rates. Mutual friction associated with kaon-vortex scattering could provide an additional dissipative mechanism, but the critical frequency curves calculated here provide a lower bound. Future related work should investigate hybrid stars with a CFL-K0 core. The role of the crust-core interface as well as the quark-hadronic matter layer should be key ingredients in obtaining constraints on the persistence of gravitational waves from stars containing superfluid quark matter in their interior.

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